RESULTS OF NUMERICAL SOLUTIONS OF NONLINEAR PROBLEMS OF HEAT CONDUCTION IN CASES OF ANNEALING IN LIQUID MEDIA

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The paper gives the results of numerical solutions of nonlinear, partly linearized, and completely linearized nonstationary problems of heat conduction in a plate annealed in a liquid medium.

The quality of annealed parts depends on the technological conditions of the hot-working, which can be optimized on the basis of an analysis of the calculated temperature fields in the parts during the annealing process. The heat-transfer problem in this case is essentially nonlinear, and therefore sufficiently accurate calculated temperature distributions can be obtained only by a numerical method [1]. Nonlinear problems are individual, by reason of the specificity of the nonlinearities themselves; therefore, any generalization requires the accumulation and analysis of solutions of many problems. Investigations in recent years have shown that the necessary engineering accuracy (errors of no more than ±1%) can be obtained when the numerical solution is carried out on analog or digital computers with due regard paid to the fairly numerous factors affecting the accuracy of the numerical solution [2-5].

An interesting fact which had apparently gone unnoticed in the past was discovered in [5] in the investigation of nonstationary temperature fields in an ingot–mold system as the ingot hardens, in a nonlinear problem with several nonlinearities. The errors in complete linearization, with specific laws governing the variation of the nonlinearities as functions of temperature, were found to be smaller than the errors resulting from partial linearization. This fact disagrees with the usual idea that errors increase as the mathematical model is simplified. This phenomenon may be called the "partial-linearization paradox." In what follows, using as an example another nonlinear problem with several nonlinearities, we shall show that a phenomenon analogous to the one observed in [5] can also occur in the annealing of objects in liquid media.

We give below the results of the numerical investigation of the following mathematical model:

$$\frac{\partial}{\partial x}\left[\lambda(T)\frac{\partial T}{\partial x}\right]-c_V(T)\frac{\partial T}{\partial \tau}=0;\ 0<x<l,\ \tau>0, \tag{1}$$

$$\lambda(T)\frac{\partial T}{\partial x}+\alpha(T_s)(T_s-T_m)=0;\ x=0, \tag{2}$$

$$\frac{\partial T}{\partial x}=0,\ x=l, \tag{3}$$

$$T(x,\ 0)=T_{max}. \tag{4}$$

Since the configuration of the metal parts is a factor affecting the temperature field but not very substantially changing the quantitative estimate of the influence of other factors, the problem was solved as a one-dimensional one. The methods used in this investigation can also be used for solving two-dimensional and three-dimensional problems in those cases in which it is necessary to take account of the complicated geometry of the parts involved. We determined the temperature fields in a plate of thickness $2l$ = 0.16 m which was annealed in

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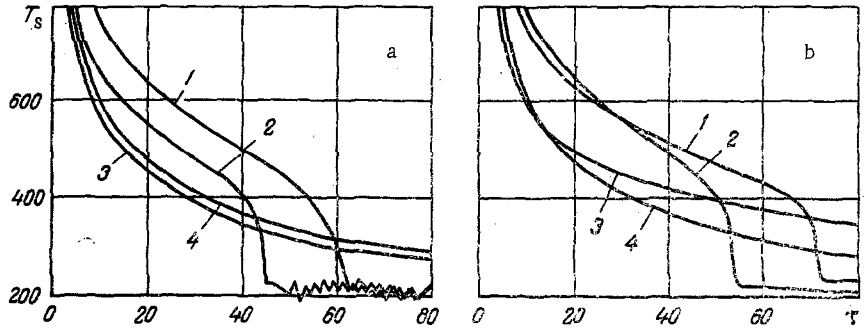

Fig. 1. Temperature at the surface of the plate (T_s, °C) as a function of time (τ, sec) obtained by solving the problem on an analog (a) and a digital (b) computer. The numbers of the curves are the numbers of the variants in the text.

water at T_m = 20°C. The characteristic sharply peaked curve of $\alpha(T_s)$ (boiling in a large volume) was taken from the data of [6], and the curves of $\lambda(T)$ and $c_V(T)$ were taken from the data of [7]. The initial values of λ, c_V, and α were given in the form of tables with a temperature interval of 25°C, T_{max} = 1400°C.

The numerical solutions for four variants of the problem were obtained by the method of networks (an implicit scheme, with a space interval of 0.01 m) on two types of computers: an analog computer, using an integrator with an electrical-model network of ohmic resistances (an R–R network), and a digital computer, the BÉSM-4. Variant 1 was the general nonlinear problem, using $\lambda(T)$, $c_V(T)$, and $\alpha(T_s)$. Variant 2 was a partially linearized problem with λ, c_V = const (for T = 500°C), and $\alpha(T_s)$. Variant 3 was a partially linearized problem with $\lambda(T)$, $c_V(T)$, and α = const (4650 W/m²·deg). Variant 4 is the completely linearized (linear) problem, with λ, c_V, α = const.

Different methods were used on the analog and the digital computers for selecting the time interval $\delta\tau$ and taking account of the nonlinearities of $\lambda(T)$, $c_V(T)$, and $\alpha(T_s)$. The interval used on the analog computer was mainly the constant value $\delta\tau$ = 1 sec. Only in a few portions of the time range was this value decreased or increased, depending on the rate of change of the surface temperature. On the digital computer the value of $\delta\tau$ was automatically selected on the basis of the curvature of the graph of $T_s(\tau)$ at the relevant stage of the calculations. On the analog computer, for each point the temperature value obtained at the preceding step in time was used for determining the values of λ, c_V, and α at the current step by a choice of the values of these quantities from the given tables, using linear interpolation. On the digital computer this was done by using the average value of the temperature at the step in question, which had been calculated with due regard to the rate of change of temperature at the preceding step. The calculations were carried out on two types of computer because the schemes for taking account of the nonlinearities and the choice of time intervals are different for analog and digital computers, and this, as will be seen later, leads to different values of the error. The choice of the type of computer in each specific case is made in accordance with a system of indicators given, for example, in [8]. In the analog and digital computer calculations we used implicit finite-difference schemes, with some modifications described above for the iteration-free (linear [9]) scheme for taking account of the nonlinearities. Such schemes have been considered in detail in [9, 10]. In the monograph [9] Samarskii made a special investigation of the errors of approximation, the stability, the convergence, and the accuracy of analog-type finite-difference schemes for quasilinear equations.

Figure 1 shows the curves obtained for the plate surface temperature as a function of time on the analog (a) and the digital (b) computers. The numbers of the curves on the figures are those of the variants listed above. Figure 2 shows the curves, as functions of time, of the linearization errors in the determination of the surface temperatures according to variants 2-4 in comparison with variant 1, constructed on the basis of the digital-computer results. The absolute error of the solution in the i-th variant was found as the difference

$$\Delta T_{si} = T_{si} - T_{s1}. \quad (5)$$

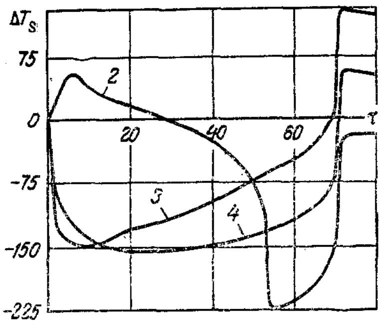

Fig. 2. Linearization errors (ΔT_S, °C) as functions of time (τ, sec) for variants 2, 3, 4, according to digital-computer results. The numbers of the curves are the numbers of the variants.

It can be seen from the figures that in problems of heat conduction in cases of annealing in liquid media we find the phenomenon noted in [5]: the errors of partial linearization are greater at certain instants of time than the errors of complete linearization. For example, at 55 sec (Fig. 2), $|\Delta T_{S2}| > |\Delta T_{S4}|$, and at 75 sec, $|\Delta T_{S3}| > |\Delta T_{S4}|$. This fact has already been encountered in the investigation of the more complicated system of mold and hardening ingot [5]. It occurs in cases in which the errors of the linearization of individual nonlinearities have opposite signs. These errors may considerably exceed the value allowable in engineering practice. For example, for variant 2 (see Figs. 1 and 2) the maximum value is $|\Delta T_{S2}| = 225$°C, which is about 50% of the instantaneous value of the temperature in variant 1. The error in the time when very rapid cooling begins is 18 sec, or about 25%. Therefore, in the choice of simpler methods of solution corresponding to the partially linearized or linear problems to which the initial general nonlinear problem is reduced, we must make a careful analysis of the effect of the nonlinearities in each specific problem.

The characteristic oscillations of curves 1 and 2 (Fig. 1a) for $x = 0$, constructed on the basis of the analog-computer results, are due in this case to the sharp changes in the value of α when there is a transition to the next step in time. Analogous oscillations arising when there are sharp changes in the boundary conditions were noted and investigated in [11-13]. They should not be confused with the oscillations caused by the physical nature of the phenomena [14]. When the time interval was selected automatically, as in the digital-computer solution of the problem, over the entire range of time variation (Fig. 1b), there were no oscillations in the surface temperature.

There is a quantitative difference between the estimates of the effects of the nonlinearities found from the analog results and those found from the digital results. For example, in the 20-80 sec time range, curve 3 in Fig. 1a is below curve 4, while in Fig. 1b curve 3 is above curve 4. The time at which very rapid cooling begins differs by 10 sec between Figs. 1a and 1b for the same variants.

Thus, in the numerical solution of problems of nonstationary heat conduction, we must obtain the necessary accuracy and keep the solution oscillation-free by an appropriate choice of the time-interval value and an appropriate scheme for taking account of the nonlinearities. The influence of the time-interval values on the accuracy has long been known, and we would not have emphasized this fact above if there were not a close connection between the value of the time interval and the scheme for taking account of the nonlinearity, on the one hand, and the nature of the law governing the variation of the nonlinearity, on the other. A time interval selected on the basis of the solution of the corresponding linear problem may yield optimal accuracy for that problem, but the same value may lead to inadmissible errors in the solution of the nonlinear problem. The choice of the time interval must be made by solving the nonlinear problem as a control problem (this may be one-dimensional) simultaneously with the choice of the scheme for taking account of the nonlinearity. Disregarding this condition may lead to incorrect quantitative, and even qualitative, estimates. If the nonlinearities are closely dependent on temperature, then for the numerical solution of such problems on analog computers we should carefully select the value of the time interval at each step of the solution, while on digital computers we should use an automatic choice of the time interval. Without control solutions of general nonlinear, partially linearized, and linear problems, we cannot draw any conclusions as to whether partial or complete linearization would be best for a given type of problem. Only a solution of the general nonlinear problem can guarantee that we will obtain results with an error of no more than ±1%.

NOTATION

T, temperature; λ, thermal conductivity; c, specific mass heat capacity; ρ, density; $c_V = c\rho$, specific volumetric heat capacity; α, heat-transfer coefficient; $2l$, plate thickness. Indices: s, surface; m, medium.

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HEAT TRANSFER BY NATURAL CONVECTION IN SPHERICAL GAS LAYERS

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The experimental data of [1] are correlated in the form of a dimensionless equation which is simple and sufficiently accurate for technical calculations and is applicable in the entire region covered by the experiment — up to $Ra_{dk} = 10^{10}$.

The extensive experimental data [1] on free-convection heat transfer through spherical layers of gas (air, CO_2, H_2) are of independent value and can also provide material for the verification of theoretical solutions in this region. Unfortunately, this valuable experimental material has not yet been analyzed and correlated in an appropriate manner: The correlation carried out in [2] lacks an adequate physical basis and is inconsistent with the main tenets of similarity theory.

An analysis of the conditions of similarity of motion and heat-transfer processes due to natural convection of gas in a region bounded by eccentric spherical boundaries with constant temperatures T_1 and T_2 led to a system of generalized variables for the description of heat transfer on surfaces bounding a spherical layer. In particular, the generalized relation for

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